Optical signature of topological insulators

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The axion coupling in topological insulators couples electric polarization with magnetic field, and magnetization with electric field. As a result, the usual laws of electromagnetic wave propagation are modified. We report on the Fresnel formula for the reflection of electromagnetic wave at the interface of materials with different axion couplings. The Brewster angle and the Goos-Hänchen effect are also studied. We find that, because of the axion coupling, in order to realize the Brewster-angle condition, the incident polarization should be rotated away from the plane of incidence. The maximum angle of rotation is $\pi/4$ when both materials have nearly the same refraction indices. This offers a convenient way to determine the axion angle by optical measurement.

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I. INTRODUCTION

A distinctive property of topological insulators is the existence of topologically protected edge or surface states.[1–6] These gapless modes consist of time-reversal (TR) conjugate components that always appear with the odd number of pairs.[3,4] Possible candidates of topological insulators are materials with an energy gap opened primarily because of the spin-orbit coupling. Experimentally, it has been confirmed that HgCd-HgTe quantum well with proper design is a two-dimensional topological insulator that exhibits quantized ballistic conductance through edge transport.[7] Also, angle-resolved photoemission spectroscopy measurements show that the $Bi_{1-x}Sb_x (x \sim 0.1)$ alloy is a three-dimensional strong topological insulator (henceforth the abbreviation “TI” refers specifically to this type).[8,9] Recently, other candidates that could be TIs at room temperature are being proposed and pursued vigorously.[10,11] These discoveries stimulate further exploration of the exotic properties of the TIs and their potential applications.

It is proposed that a quantized magnetoelectric coupling can be induced in a TI when a weak TR-breaking perturbation is applied.[12] This coupling is described by the following effective Lagrangian density,

$$\mathcal{L}_\Theta = \alpha \frac{\Theta}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} = \alpha \frac{\Theta}{4\pi^2} \mathbf{E}\cdot\mathbf{B}, \tag{1}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant, $F_{\mu\nu}$ is the electromagnetic (EM) field tensor, and $\tilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\lambda\delta}F_{\lambda\delta}$ is its dual tensor. Such a term is called an axion coupling since a space-time dependent Θ is known as an axion field in particle physics.[13] Its contribution to the partition function is given by e^{iS_Θ}, where the action $S_\Theta = \hbar^{-1}\int d^3x dt \mathcal{L}_\Theta$ is always an integer multiple of Θ when integrated over a closed four-dimensional space-time manifold. Therefore, the theory is invariant under the shift, $\Theta \to \Theta + 2\pi$. On the other hand, for a system with an open boundary, such a 2π shift results in an addition of a quantum Hall conductance for the surface state. Under TR transformation, $\Theta \to -\Theta$ such that Eq. (1) can be TR invariant. As a result, TR invariance allows a nontrivial Θ value that is an odd integer multiple of π, such is the case for a TI.[12]

Through the magnetoelectric coupling, an applied electric field can induce magnetization, while a magnetic field can induce polarization. Therefore, unusual EM properties can emerge in these materials. For example, a tangential electric field on the surface of a TI would generate a transverse surface current, giving rise to a half-integer quantum Hall effect.[3,4,12–14] In addition, a point charge near the surface of a TI would induce a vortex current (because of the tangential component of the electric field produced by the point charge), which generates a magnetic field that can be simulated by an image monopole below the surface.[15]

The axion effect can be probed by optical methods, e.g., via the Faraday rotation of a transmitted EM wave.[12] The estimated maximum angle of rotation (for normal incidence) is 10^{-3} rad, which is small but within experimental reach. In the present work, we examine the optical signatures of TI in detail and propose several ways to determine the strength of the axion coupling. It is found that the Fresnel formula governing the intensity of reflection wave is modified. Also, the polarization-preserving eigenmodes in reflection are no longer the ordinary ones with parallel (TM) and perpendicular (TE) states of polarization. The new basis of polarization rotates away from the usual TM and TE modes. Such a rotation can be significant under proper conditions, from which one can determine the strength of axion coupling easily. Furthermore, we find that, even though the Snell's law and the critical angle for total internal reflection remain unchanged for the TI, the Brewster angle and the Goos-Hänchen (GH) shift[16] are modified by the axion coupling.

The calculations reported here can also be applied to magnetoelectric materials (such as Cr_2O_3) and polarized insulators with similar axion couplings (but $\Theta \neq \pi$),[17,18] if other tensorial components of the EM susceptibility do not complicate the optical signatures.

II. FRESNEL FORMULAS

The EM equations with axion coupling take the usual Maxwellian forms. One only has to modify the electric dis-

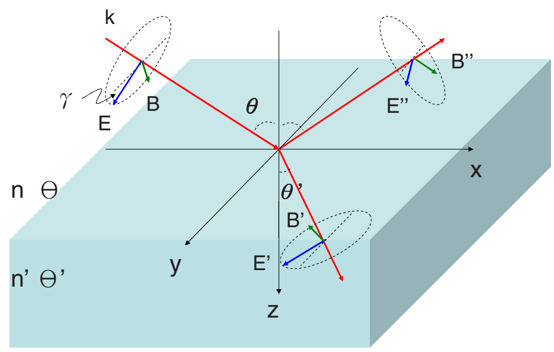

FIG. 1. (Color online) Optical reflection and refraction at the interface between two materials with refraction indices n and n' and axion angles Θ and Θ'.

placement **D** and magnetic field **H** as follows:[13,15,19] $\mathbf{D}=\epsilon\mathbf{E}+\alpha(\Theta/\pi)\mathbf{B}$ and $\mathbf{H}=(\mathbf{B}/\mu)-\alpha(\Theta/\pi)\mathbf{E}$. As a result, two of the Maxwell equations, the Coulomb's law and the Ampere's law are modified. Since the other two source-free equations remain unchanged, the fields **E** and **B** and the wave vector **k** remain orthogonal to each other even if $\Theta\neq 0$.

One can show that the boundary conditions of an interface without surface charge and surface current take the usual forms. Therefore, the normal components of **D** and **B**, as well as the tangential components of **E** and **H**, have to be continuous across the interface as usual. The optical reflection and refraction at the interface between two materials (one or both can be TIs) is shown in Fig. 1. By matching the phases $\exp(i\mathbf{k}\cdot\mathbf{x})$ of the EM plane waves, it is not difficult to prove that the angle of reflection is equal to the angle of incidence θ, and the Snell's law remains valid, even if Θ is not zero on either or both sides.

With the help of the boundary conditions, one gets the relations between the incident field and the reflection field. Let $E_\perp$ and $E_\parallel$ be the components of the incident wave along TE and TM states of polarization, respectively, then the Fresnel equation for the reflected wave (with components $E''_\perp$ and $E''_\parallel$) can be written in a matrix form[20]

$$\begin{pmatrix} E''_\perp \\ E''_\parallel \end{pmatrix} = \frac{1}{\Delta}\begin{bmatrix} (n^2-n'^2-\bar{\alpha}^2)+nn'\xi_- & 2\bar{\alpha}n \\ 2\bar{\alpha}n & -(n^2-n'^2-\bar{\alpha}^2)+nn'\xi_- \end{bmatrix} \times\begin{pmatrix} E_\perp \\ E_\parallel \end{pmatrix}, \tag{2}$$

where $\bar{\alpha}=\alpha(\Theta'-\Theta)/\pi$, $\Delta=n^2+n'^2+\bar{\alpha}^2+nn'\xi_+$, and

$$\xi_\pm \equiv \frac{\cos\theta}{\cos\theta'} \pm \frac{\cos\theta'}{\cos\theta}. \tag{3}$$

For materials with relative magnetic susceptibility μ, these relations and those following from them in the discussion below are still valid if one replaces n and n' by n/μ and n'/μ', respectively.[21]

Because the matrix relating the reflection field to the incident one is real and symmetric, it can be diagonalized by means of an orthogonal transformation. That is, after taking the new orthogonal basis of polarization as $\hat{e}_1=(\cos\gamma,\sin\gamma)^T$ and $\hat{e}_2=(-\sin\gamma,\cos\gamma)^T$, in which

$$\tan(2\gamma)=\frac{2\bar{\alpha}n}{n^2-n'^2-\bar{\alpha}^2}, \tag{4}$$

the Fresnel equation for the reflected wave reduces to a diagonal form

$$E''_{1,2}=r_{1,2}\;E_{1,2}, \tag{5}$$

with

$$r_1=\frac{\bar{n}\cos\theta-\bar{n}'\cos\theta'}{\bar{n}\cos\theta+\bar{n}'\cos\theta'},$$

$$r_2=\frac{\bar{n}'\cos\theta-\bar{n}\cos\theta'}{\bar{n}'\cos\theta+\bar{n}\cos\theta'}. \tag{6}$$

Here E''_1 and E''_2 (E_1 and E_2) are the components of the reflected (incident) wave along the directions of $\hat{e}_1$ and $\hat{e}_2$, respectively. The *effective* indices of refraction $\bar{n}$ and $\bar{n}'$ are defined as

$$\bar{n}=\frac{1}{2}\left[\sqrt{(n+n')^2+\bar{\alpha}^2}+e^{i\delta}\sqrt{(n-n')^2+\bar{\alpha}^2}\right],$$

$$\bar{n}'=\frac{1}{2}\left[\sqrt{(n+n')^2+\bar{\alpha}^2}-e^{i\delta}\sqrt{(n-n')^2+\bar{\alpha}^2}\right], \tag{7}$$

with $e^{i\delta}=(n^2-n'^2-\bar{\alpha}^2)/|n^2-n'^2-\bar{\alpha}^2|$. Thus the reflection for the two directions of polarization along $\hat{e}_1$ and $\hat{e}_2$ becomes decoupled.

In the limit of $\bar{\alpha}=0$, the new basis of $\hat{e}_1$ and $\hat{e}_2$ reduces to the original one of $\hat{e}_\perp$ and $\hat{e}_\parallel$, and the effective indices of refraction $\bar{n}$ and $\bar{n}'$ become n and n', respectively. In general, the axion effect is tiny since $\bar{\alpha}^2$ is about six orders of magnitude smaller than n^2. In Fig. 2, we show the change in reflectance due to the axion effect, in which $\delta r_1^2\equiv(r_1)^2|_{\bar{\alpha}\neq 0}-(r_1)^2|_{\bar{\alpha}=0}$ (similarly for δr_2^2). As one can see from Fig. 2, even though the change can be enhanced near the critical angle, the effect remains tiny and may be difficult to detect.

However, one can see from Eq. (4) that the angle of rotation γ of the new basis of polarization can be significant when $n\simeq n'$ (see Fig. 3). This provides a convenient probe of the axion coupling when combined with the measurement of the Brewster angle, which is possible only if the incident polarization is along the $\hat{e}_2$ direction (see below). For example, given $n=10$ (for materials such as $Bi_{1-x}Sb_x$), if one can find a usual insulator with $n'=9$, then the angle γ of rotation is on the order of 0.1 degree. In the ideal limit with $n\to\sqrt{n'^2+\bar{\alpha}^2}$, one can see that γ reaches its maximum value of $\pm\pi/4$.[22]

III. BREWSTER ANGLE

Since the Fresnel reflection coefficients shown in Eq. (6) are similar to those in the usual cases, one expects the re-

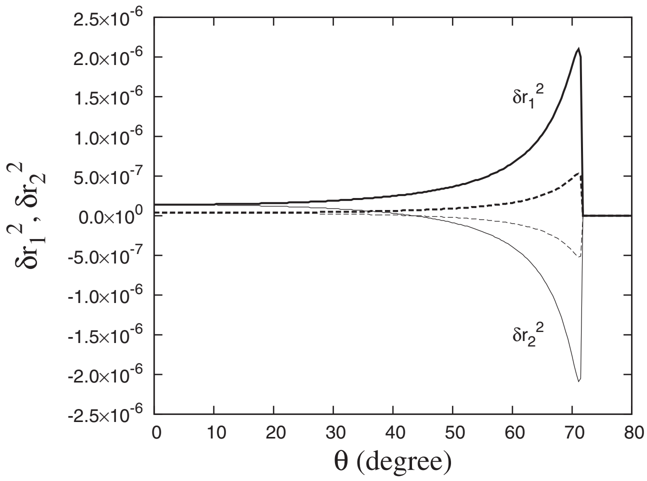

FIG. 2. Changes in reflectance δr_1^2 and δr_2^2 (defined in the text) due to the axion coupling, plotted as a function of the incident angle θ. Solid lines are for $\Theta'-\Theta=\pi$; dashed lines are for $\Theta'-\Theta=\pi/2$ (bold solid and dashed lines are for δr_1^2). The critical angle $\theta_c=\sin^{-1}(n'/n)=71.805°$ for $(n,n')=(10,9.5)$.

flected wave to vanish when a given linearly polarized wave is incident at some Brewster angle. It is not difficult to realize that the eigenmode of polarization along $\hat{e}_2$ plays the role of the usual TM wave. By using the Snell's law, $n\sin\theta=n'\sin\theta'$, and requiring $r_2=0$, one obtains the Brewster angle θ_B satisfying

$$\tan(\theta_B)=\left(\frac{n'}{n}\right)\sqrt{\left|\frac{1-(\bar{n}'/\bar{n})^2}{1-(n'/n)^2}\right|}. \qquad (8)$$

We note that, when $\bar{\alpha}$ is nonzero, the usual relation $\tan(\theta_B)=n'/n$ is not valid. This implies that the direction of the reflected beam is *no longer* perpendicular to that of the refracted beam. With the Brewster-angle condition, one can determine the $\hat{e}_2$ direction accurately, and the strength of the axion coupling can thus be obtained using Eq. (4).

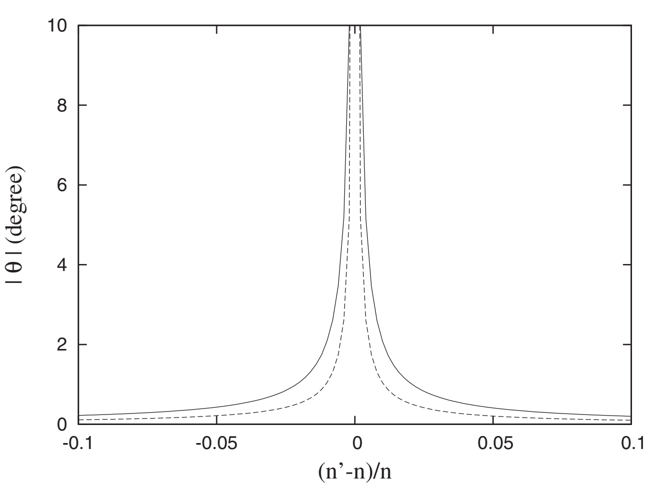

FIG. 3. Rotation angle γ of the eigenmodes plotted as a function of $(n'-n)/n$, where n and n' are the refraction indices on the two sides of the interface. Solid lines are for $\Theta'-\Theta=\pi$; dashed lines are for $\Theta'-\Theta=\pi/2$.

IV. GOOS-HÄNCHEN EFFECT

Since the Snell's law remains unchanged, when $n>n'$ and the angle of incidence $\theta>\theta_c=\sin^{-1}(n'/n)$, total internal reflection occurs as usual. The Fresnel reflection coefficients for the two eigenmodes in Eq. (6) now become complex, $r_{1,2}=\exp(i\phi_{1,2})$, where the phase angles $\phi_{1,2}$ satisfy the following relations

$$\tan\frac{\phi_1}{2}=-\frac{\bar{n}'}{\bar{n}}\frac{1}{\sin\theta_c}\frac{\sqrt{\sin^2\theta-\sin^2\theta_c}}{\cos\theta}, \qquad (9)$$

$$\tan\frac{\phi_2}{2}=-\frac{\bar{n}}{\bar{n}'}\frac{1}{\sin\theta_c}\frac{\sqrt{\sin^2\theta-\sin^2\theta_c}}{\cos\theta}. \qquad (10)$$

The general expression of the GH shift (the longitudinal shift in the reflected beam during total internal reflection) is known as $D=-(\lambda/2\pi)(d\phi/d\theta)$,[23] in which λ is the wavelength in the medium of higher index of refraction. Thus the longitudinal displacements of the reflected beams for the two eigenstates of plane polarization become

$$D_1=D_0\frac{(\bar{n}'/\bar{n})\sin\theta_c\cos^2\theta_c}{[(\bar{n}'/\bar{n})^2-\sin^2\theta_c]\sin^2\theta+[1-(\bar{n}'/\bar{n})^2]\sin^2\theta_c}, \qquad (11)$$

$$D_2=D_0\frac{(\bar{n}'/\bar{n})\sin\theta_c\cos^2\theta_c}{[1-(\bar{n}'/\bar{n})^2\sin^2\theta_c]\sin^2\theta-[1-(\bar{n}'/\bar{n})^2]\sin^2\theta_c}. \qquad (12)$$

Here,

$$D_0=\frac{\lambda}{\pi}\frac{\sin\theta}{\sqrt{\sin^2\theta-\sin^2\theta_c}} \qquad (13)$$

gives the GH shift in the case of $\Theta=0$ for both media and with perpendicular incident state of polarization. Typically

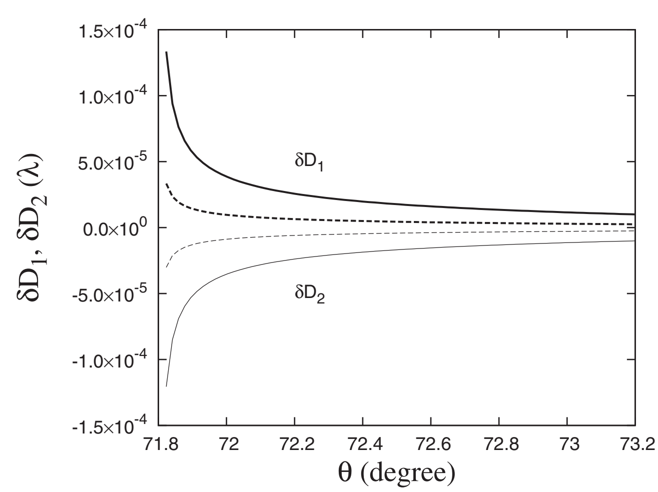

FIG. 4. Changes in the GH shift δD_1 and δD_2 (defined in the text) due to the axion coupling, plotted as a function of the incident angle θ. Solid lines are for $\Theta'-\Theta=\pi$; dashed lines are for $\Theta'-\Theta=\pi/2$ (bold solid and dashed lines are for δD_1). The critical angle $\theta_c=71.805°$ for $(n,n')=(10,9.5)$.

the GH shift is on the order of wavelength. However, near the critical angle θ_c, the GH shift, along with the axion-induced shift, can be amplified significantly. The axion-induced GH shift $\delta D_1 \equiv D_1|_{\bar{\alpha}\neq 0} - D_1|_{\bar{\alpha}=0}$ (similarly for δD_2) is shown in Fig. 4. One can see that the values diverge near the critical angle. However, since the original GH shift also diverges there, the relative change remains small. Nevertheless, one may employ a TI wire with multiple total reflections to enhance the induced shifts and make them easily measurable.[24]

In summary, because of the importance of the axion coupling in TI, we propose several optical methods to measure the Θ angle of TI and other magnetoelectric materials. It should be noted that, in practice, a magnetic overlayer may be required to open an energy gap at the Dirac point of the surface states, which could complicate axion's optical signatures. Also, both the Faraday-rotation method in Ref. 12 and the method in this Brief Report require an electromagnetic wave with microwave frequency or lower, in order not to induce interband transitions across the energy gap, which would destroy the effective axion coupling. To enhance the optical effects, one should match (interface) the TI with a material with similar refraction index. The easiest way to observe the axion effect is to measure the rotation angle of the eigenmode with the help of the Brewster angle. Such an ellipsometry measurement not only can confirm the existence of the axion electrodynamics in TIs, but can also determine the axion-coupling strength.

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